Backpaper examination 2010
M.Math.Ist year
Differential geometry I
Instructor — B.Sury
Answer any SIX

Q 1.
Consider a smooth function f from $\mathbf{R}$ to itself which satisfies $f(t) = 0$ for $t \leq 0$ and $f(t) > 0$ for $t > 0$. Consider the curve $\alpha(t) = (t, f(t), f(-t))$. Find all points on the curve where the curvature is zero.

Q 2.
Show that the curve $\alpha(t) = (3t - t^3, 3t^2, 3t + t^3)$ is not a helix but that $\tau = k$ at each point.

Q 3.
(i) Show that the cylinder $S = \{(x, y, z) : x^2 + y^2 = 1\}$ can be covered by a single parametrization/patch.
(ii) Prove also that the sphere cannot be covered by a single parametrization/patch.

Q 4.
Prove that applying a rigid motion (that is, rotation and translation) of $\mathbf{R}^3$ to a local parametrization of a surface S does not change the first fundamental form.

Q 5.
Consider a tangent developable S of a curve α of unit speed; that is, a parametrization is $f(u, v) = \alpha(u) + v\alpha'(u)$. Show that the first fundamental form is $(1 + v^2K^2)du^2 + 2dudv + dv^2$.

Q 6.
Prove that the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is an orientable surface. Try to prove this by viewing the ellipsoid as a level surface (that is, without writing out local parametrizations).

Q 7.
Prove that the geodesics on a sphere are great circles.